R. CLIFTON BAILEY
STATISTICS SEMINAR SERIES 2016-2017

Affine Urn Models

Hosam Mahmoud

Department of Statistics
George Washington University

Johnson Center 326 – Meeting Room B
4000 University Drive, Fairfax, VA 22030

October 6, 2017
Time: 11:00 A.M. - 12:00 Noon

Abstract:

We study a class of *balanced* urn schemes on balls of two colors (say white and blue). At each drawing, a sample of size $m \geq 1$ is taken out from the urn, and ball addition rules are applied. We consider these multiple drawings under sampling with or without replacement. We further classify ball addition matrices according to the structure of the expected value into affine and nonaffine classes. We give a necessary and sufficient condition for a scheme to be in the affine subclass, for which we get explicit results for the expected value and second moment of the number of white balls after n steps and an asymptotic expansion of the variance. Moreover, we uncover a martingale structure. This unifies several earlier works focused on special cases of urn models with multiple drawings as well as the special case of drawing one ball in each sample ($m = 1$). The class is parametrized by Λ, specified by the ratio of the two eigenvalues of a "reduced" ball replacement matrix and the sample size. We categorize the class into small-index urns ($\Lambda < 1/2$) critical-index urns ($\Lambda = 1/2$), and large-index urns ($\Lambda > 1/2$), and triangular urns. We obtain central limit theorems for small- and critical index urns and prove almost-sure convergence for triangular and large-index urns. Moreover, we discuss the moment structure of large-index urns and triangular urns.

R. CLIFTON BAILEY
STATISTICS SEMINAR SERIES 2016-2017